Chapter Two

Set and Function

The word 'set' is familiar to us, such as dinner set, set of natural numbers, set of rational numbers etc. As a modern weapon of mathematics, the use of set is extensive. The German mathematician George Cantor (1844 – 1918) first explained his opinion about set. He created a sensation in mathematics by expressing the idea of infinite set and his conception of set is known as 'set theory'. In this chapter, the main objectives are to solve problems through using mathematics and symbols from the conception of set and to acquire knowledge about function.

At the end of this chapter, the students will be able to :

- Explain the conception of set and subset and express them by symbols
- Describe the method of expressing set
- Explain the infinite set and differentiate between finite and infinite set
- Explain and examine the union and the intersection of set
- Explain power set and form power set with two or three elements
- Explain ordered pair and cartesian product
- Prove the easy rules of set by example and Venn Diagram and solve various problems using the rules of set operation
- Explain and form sets and functions
- Explain what are domain and range
- Determine the domain and range of a function
- Draw the graph of the function.

Set

Well defined assembling or collection of objects of real or imaginative world is called sets, such as, the set of three textbook of Bangla, English and Mathematics, set of first ten natural odd numbers, set of integers, set of real numbers etc.

Set is generally expressed by the capital letters of english alphabets, $A, B, C, ...X, Y, Z$. For example, the set of three numbers 2, 4, 6 is $A = \{2, 4, 6\}$

Each object or member of set is called set element. Such as, if $B = \{a, b\}$, a and b are elements of B. The sign of expressing an element is '$\in$'.

$\therefore$ $a \in B$ and read as a belongs to B

$b \in B$ and read as b belongs to B

no element c is in the above set B.

$\therefore\ c \notin B$ and read as c does not belong to B.

Method of describing Sets :

Set is expressed in two methods :(1) Roster Method or Tabular Method and (2) Set Builder Method.

(1) Tabular Method : In this methods, all the set elements are mentioned particularly by enclosing them within second bracket { } and if there is more than one element, the elements are separated by using a comma (,).

For example : $A = \{a, b\}$, $B = \{2, 4, 6\}$, $C =$ {Niloy, Tisha, Shuvra} etc.

(2) Set Builder Method : In this methods, general properties are given to determine the set element, without mentioning them particularly :

Such as, $A = \{x : x$ is a natural odd number}, $B = \{x : x$ denotes the first five students of class IX} etc.

Here, by ':' 'such as' or in short 'such that' is indicated. Since in this method, set rule or condition is given to determining the set elements of this method, is called rule method.

Example 1. Express the set $A = \{7, 14, 21, 28\}$ by set builder method.

Solution : The elements of set A are $7, 14, 21, 28$

Here, each element is divisible by 7, that is, multiple of 7 and not greater than 28.

$\therefore\ A = \{x : x$ multiple of 7 and $x \leq 28\}$.

Example 2. Express the set $B = \{x : x,$ factors of 28} by tabular method.

Solution : Here, $28 = 1 \times 28$

$= 2 \times 14$

$= 4 \times 7$

$\therefore$ factors of 28 are $1, 2, 4, 7, 14, 28$

Required set $B = \{1, 2, 4, 7, 14, 28\}$

Example 3. Express $C = \{x : x$ is a positive integer and $x^2 < 18\}$ by tabular method.

Solution : Positive integers are $1, 2, 3, 4, 5, ...$

Here if $x = 1,\ x^2 = 1^2 = 1$

if $x = 2,\ x^2 = 2^2 = 4$

if $x = 3,\ x^2 = 3^2 = 9$

if $x = 4,\ x^2 = 4^2 = 16$

if $x = 5,\ x^2 = 5^2 = 25$; which is greater than 18.

$\therefore$ The acceptable positive integers by the condition are $1, 2, 3, 4$

$\therefore$ Required set is $C = \{1, 2, 3, 4\}$

Activity : 1. Express the set $C = \{-9, -6, -3, 3, 6, 9\}$ by set builder method.
2. Express the set $Q = \{y : y$ is an integer and $y^3 \leq 27\}$ by tabular method.

Finite set : The set whose numbers of elements can be determined by counting is called finite set. For example, D = $\{x, y, z\}$, E = $\{3, 6, 9, ..., 60\}$, $F = \{x : x$ is a prime number and $30 < x < 70\}$ etc. are finite set. Here D set has 3 elements, E set has 20 elements and F set has 9 elements.

Infinite set : The set whose numbers of elements can not be determined by counting is called infinite set. For example :, $A = \{x : x$ is natural odd numbers$\}$, set of natural number $N = \{1, 2, 3, 4, ...\}$, set of integers $Z = \{..... -3, -2, -1, 0, 1, 2, 3.......\}$, set of rational numbers $Q = \left\{\frac{P}{q} : p\right.$ and q is as integer and $q \neq 0\}$, set of real numbers = R etc. are infinite set.

Example 4. Show that the set of all natural numbers is an infinite set.

Solution : Set of natural number $N = \{1, 2, 3, 4, 5, 6, 7, 8, ...\}$

Taking odd natural numbers from set N, the formed set $A = \{1, 3, 5, 7, ...\}$

,, even ,, ,, ,, ,, N, the formed set $B = \{2, 4, 6, 8, ...\}$

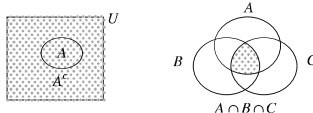
The set of multiple of 3 $C = \{3, 6, 9, 12, ...\}$ etc.

Here, the elements of the set A, B, C formed from set N can not be determined by counting. So A, B, C is an infinite set.

$\therefore$ N is an infinite set.

Activity : Write the finite and infinite set from the sets given below :
1. $\{3, 5, 7\}$ 2. $\{1, 2, 2^2, ...2^{10}\}$ 3. $\{3, 3^2, 3^3, ...\}$ 4. $\{x : x$ is an integer and $x < 4\}$
5. $\left\{\frac{p}{q} : p\right.$ and q are co-prime and $q > 1\}$ 6. $\{y : y \in N$ and $y^2 < 100 < y^3\}$.

Empty set : The set which has no element is called empty set. Empty set is expressed by $\{\}$ or ϕ. Such as, set of three male students of Holycross school $\{x \in N : 10 < x < 11\}$, $\{x \in N : x$ is a prime number and $23 < x < 29\}$ etc.

Venn-Diagram : John Venn (1834-1883) introduced set activities by diagram. Here the geometrical figure on the plane like rectangular area, circular area and triangular area are used to represent the set under consideration. These diagrams are named Venn diagram after his name.

Subset : $A = \{a, b\}$ is a set. By the elements of set A, the sets $\{a, b\}$, $\{a\}$, $\{b\}$ can be formed. Again, by not taking any element ϕ set can be formed.

Here, each of $\{a, b\}$, $\{a\}$, $\{b\}$, ϕ is subset of set A.

So, the number of sets which can be formed from any set is called subset of that set. The sign of subset is $\subset$. If B is the subset of A, it is read as $B \subset A$. B is a subset of A. From the above subsets, $\{a, b\}$ set is equal to A.

$\therefore$ Each set is the subset of itself.

Again, from any set, ϕ set can be formed.

$\therefore$ ϕ is a subset of any set.

$Q = \{1, 2, 3\}$ and $R = \{1, 3\}$ are two subsets of $P = \{1, 2, 3\}$. Again $P = Q$

$\therefore$ $Q \subseteq P$ and $R \subset P$.

Proper Subset :

If the number of elements of any subset formed from a set is less than the given set, it is called the proper subset. For example : $A = \{3, 4, 5, 6\}$ and $B = \{3, 5\}$ àre two sets. Here, all the elements of B exist in set A. $\therefore$ $B \subset A$

Again, the number of elements of B is less then the number of elements of A.

$\therefore$ B is a proper subset of A and expressed as $B \subseteq A$.

Example 5. Write the subsets of $P = \{x, y, z\}$ and find the proper subset from the subsets.

Solution : Given, $P = \{x, y, z\}$

Subsets of P are $\{x, y, z\}$, $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x\}$, $\{y\}$, $\{z\}$, ϕ.

Proper subsets of P are $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x\}$, $\{y\}$, $\{z\}$

Equivalent Set :

If the elements of two or more sets are the same, they are called equivalent sets. Such as, $A = \{3, 5, 7\}$ and $B = \{5, 3, 7\}$ are two equal sets and written as $A = B$.

Again, if $A = \{3, 5, 7\}$, $B = \{5, 3, 3, 7\}$ and $C = \{7, 7, 3, 5, 5\}$, the sets A, B and C are equivalent. That is, $A = B = C$

It is to be noted if the order of elements is changed or if the elements are repeated, there will be no change of the set.

Difference of Set : Suppose, $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 5\}$. If the elements of set B are discarded from set A, the set thus formed is $\{1, 2, 4\}$ and written as $A \setminus B$ or $A - B = \{1, 2, 3, 4, 5\} - \{3, 5\} = \{1, 2, 4\}$

So, if a set is discarded from any set, the set thus formed is called different set.

Example 6. If $P = \{x : x$, factors of $12\}$and $Q = \{x : x$, multiples of 3 and $x \leq 12\}$, find $P - Q$.

Solution : Given, $P = \{x : x$, factors of $12\}$

Here, factors of 12 are $1, 2, 3, 4, 6, 12$

$\therefore\ P = \{1, 2, 3, 4, 6, 12\}$

Again, $Q = \{x : x$, multiple of 3 and $x \leq 12\}$

Here, multiple of 3 upto 12 are $3, 6, 9, 12$

$\therefore\ Q = \{3, 6, 9, 12\}$

$\therefore\ P - Q = \{1, 2, 3, 4, 6, 12) - \{3, 6, 9, 12\} = \{1, 2, 4\}$

Required set $\{1, 2, 4\}$

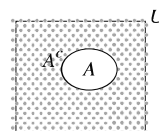
Universal Set :

All sets related to the discussions are subset of a definite set. Such as, $A = \{x, y\}$ is a subset of $B = \{x, y, z\}$. Here, set B is called the universal set in with respect to the set A.

So, if all the sets under discussion are subsets of a particular set, that particular set is called the universal set with respect to its subsets.

Universal set is generally expressed by U. But universal set can be expressed by other symbols too. Such as, if set of all even natural numbers $C = \{2, 4, 6........\}$ and set of all natural numbers $N = \{1, 2, 3, 4,...........\}$, the universal set with respect to C will be N.

Complement of a Set :

U is an universal set and A is the subset of U. The set formed by all the elements excluding the elements of set A is called the complement of set A. The complement of the set A is expressed by A^c or A'. Mathematically, $A^c = U \setminus A$

Let, P and Q are two sets and the elements of Q which are not elements of P are called complement set of Q with respect to P and written as $Q^c = P \setminus Q$.

Example 7. If $U = \{1, 2, 3, 4, 6, 7\}$, $A = \{2, 4, 6, 7\}$ and $B = \{1, 3, 5\}$, determine A^c and B^c.

Solution : $A^c = U \setminus A = \{1, 2, 3, 4, 6, 7\} \setminus \{2, 4, 6, 7\} = \{1, 3, 5\}$

and $B^c = U \setminus B = \{1, 2, 3, 4, 6, 7\} \setminus \{1, 3, 5\} = \{2, 4, 6, 7\}$

∴ Required set $A^c = \{1, 3, 5\}$ and $B^c = \{2, 4, 6, 7\}$

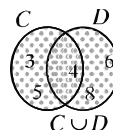
Union of Sets :

The set formed by taking all the elements of two or more sets is called union of sets. Let, A and B are two sets. The union of A and B set is expressed by $A \cup B$ and read as A union B. In the set builder method $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

Example 8. If $C = \{3, 4, 5\}$ and $D = \{4, 6, 8\}$, determine $C \cup D$.

Solution : Given that, $C = \{3, 4, 5\}$ and $D = \{4, 6, 8\}$

$\therefore\ C \cup D = \{3, 4, 5\} \cup \{4, 6, 8\} = \{3, 4, 5, 6, 8\}$

$C \cup D$

Intersection of Sets:

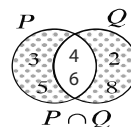
The set formed by the common elements of two or more sets is called intersection of sets. Let, A and B are two sets. The intersection of A and B is expressed by $A \cap B$ and read as A intersection B. In set building method, $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Example 9. If $P = \{x \in N : 2 < x \leq 6\}$ and $Q = \{x \in N : x$ are even numbers and $x \leq 8\}$, find $P \cap Q$.

Solution : Given that, $P = \{x \in N : 2 < x \leq 6\} = \{3, 4, 5, 6\}$

and $Q = \{x \in N : x$ are even numbers and $x \leq 8\} = \{2, 4, 6, 8\}$

$\therefore\ P \cap Q = \{3, 4, 5, 6\} \cap \{2, 4, 6, 8\} = \{4, 6\}$

∴ Required set is $\{4, 6\}$

Disjoint Sets:

If there is no common element in between two sets, the sets are disjoined sets. Let , A and B are two sets. If $A \cap B = \phi$, A and B will be mutually disjoint sets.

Activity : If $U = \{1, 3, 5, 7, 9, 11\}$, $E = \{1, 5, 9\}$ and $F = \{3, 7, 11\}$, find, $E^c \cup F^c$ and $E^c \cap F^c$.

Power Sets :

$A = \{m, n\}$ is a set. The subsets of A are $\{m, n\}$, $\{m\}$, $\{n\}$, ϕ. Here, the set of subsets $\{\{m, n\}, \{m\}, \{n\}, \phi\}$ is called power set of set A. The power set of A is expressed as $P(A)$. So, the set formed with all the subsets of any set is called the power set of that set.

Example 10. $A = \{\}$, $B = \{a\}$, $C = \{a, b\}$ are three sets.

Here, $P(A) = \{\phi\}$

$\therefore$ The number of elements of set A is 0 and the number of elements of its power set $= 1 = 2^0$

Again, $P(B) = \{\{a\}, \phi\}$

$\therefore$ The number of elements of set B is 1 and the number of elements of its power set is $= 2 = 2^1$

and $P(C) = \{\{a, b\}, \{a\}, \{b\}, \phi\}$

$\therefore$ The number of elements of set C is 2 and the number of elements of its power set is $= 4 = 2^2$

So, if the number of elements is n of a set, the number of elements of its power set will be 2^n.

Activity : If $G = \{1, 2, 3\}$, determine $P(G)$ and show that the number of elements of $P(G)$ supports 2^n.

Ordered pair :

Amena and Sumona of class VIII stood 1st and 2nd respectively in the merit list in the final examination. According to merit they can be written (Amena, Sumona) as a pair. In this way, the pair fixed is an ordered pair.

Hence, in the pair of elements, the first and second places are fixed for the elements of the pair and such expression of pair is called ordered pair.

If the first element of an ordered pair is x and second element is y, the ordered pair will be (x, y). The ordered pair (x, y) and (a, b) will be equal or $(x, y) = (a, b)$ if $x = a$ and $y = b$.

Example 11. If $(2x + y, 3) = (6, x - y)$, find (x, y).

Solution : Given that, $(2x + y, 3) = (6, x - y)$

According to the definition of ordered pair, $2x + y = 6$........(1)

and $x - y = 3$.........(2)

Adding equation (1) and (2), we get, $3x = 9$ or, $y = 3$

Putting the value of x in equation (1), we get, $6 + y = 6$ or, $y = 0$

$\therefore$ $(x, y) = (3, 0)$.

Cartesian Product :

Wangsu decided to give layer of white or blue colour in room of his house at the inner side and red or yellow or green colour at the outer side. The set of colour of inner wall $A =$ {white, blue} and set of colour of outer wall $B =$ {red, yellow,

green} Wangsu can apply the colour of his room in the form of ordered pair as (white, red), (white, yellow), (white, green), (blue, red), (blue, yellow), (blue, green).
The given ordered pair is written as.
$A\times B=\{$(white, red), (white, yellow), (white, green), (blue, red), (blue, yellow), (blue, green)$\}$
This is the cartesian product set.
In set builder method, $A\times B=\{(x,y);x\in A$ and $y\in B\}$
$A\times B$ is read as A *cross* B.

Example 12. If $P=\{1,2,3\}$, $Q=\{3,4\}$ and $R=P\cap Q$, determine $P\times R$ and $R\times Q$.

Solution : Given that, $P=\{1,2,3\}$, $Q=\{3,4\}$
and $R=P\cap Q=\{1,2,3\}\cap\{3,4\}=\{3\}$
$\therefore\ P\times R=\{1,2,3,\}\times\{3\}=\{(1,3).(2,3),(3,3)\}$
and $R\times Q=\{3\}\times\{3,4\}=\{(3,3),(3,4)\}$

Activity : 1. If $\left(\frac{x}{2}+\frac{y}{3},1\right)=\left(1,\frac{x}{3}+\frac{y}{2}\right)$, find (x,y).
2. If $P=\{1,2,3\}$, $Q=\{3,4\}$ and $R=\{x,y\}$, find $(P\cap Q)\times R$ and $(P\cap Q)\times Q$.

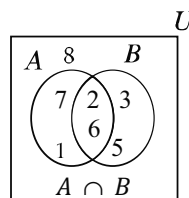
Example 13. Find the set where 23 is reminders in each case when 311 and 419 are divided by the natural numbers.

Solution : The numbers when 311 and 419 are divided by rational numbers and 23 is reminder, will be greater than 23 and will be common factors of 311 − 23 = 288 and 419 − 23 = 396.
Let, the set of factors of 288 greater than 23 is A and the set of factors of 396 is B.
Here,
$288=1\times288=2\times144=3\times96=4\times72=6\times48=8\times36=9\times32=12\times24=16\times18$
$\therefore\ A=\{24,32,36,48,72,96,144,288\}$
Again,
$396=1\times396=2\times198=3\times132=4\times99=6\times66=9\times44=11\times36=12\times33=18\times22$
$\therefore\ B=\{33,36,44,66,99,132,198,396\}$
$\therefore\ A\cap B=\{24,32,36,48,72,96,144,288\}\cap\{33,36,44,66,99,132,198,396\}=\{36\}$
Required set is $\{36\}$

Example 14. If $U=\{1,2,3,4,5,6,7,8\}$, $A=\{1,2,6,7\}$, $B=\{2,3,5,6\}$ and $C=\{4,5,6,7\}$, show that, (*i*) $(A\cup B)'=A'\cap B'$ and (*ii*) $(A\cap B)\cup C=(A\cup C)\cap(B\cup C)$

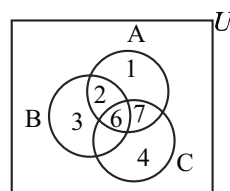
Solution : (*i*)

In the figure, U by rectangle and the sets of A and B are denoted by two mutually intersecting circle sets.

set	Elements
$A \cup B$	1, 2, 3, 4, 5, 6, 7
$(A \cup B)'$	4, 8
A'	3, 4, 5, 8
B'	1, 4, 7, 8
$A' \cap B'$	4, 8

$\therefore (A \cup B)' = A' \cap B'$

Solution : (ii) In figure, U by rectangle and sets of A, B, C are denoted by three mutually intersecting circles.

Observe ,

Set	Elements
$A \cap B$	2, 6
$(A \cap B) \cup C$	2, 4, 5, 6, 7
$A \cup C$	1, 2, 4, 5, 6, 7
$B \cup C$	2, 3, 4, 5, 6, 7
$(A \cap C) \cap (B \cup C)$	2, 4, 5, 6, 7

$\therefore (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Example 15. Among 100 students, 92 in Bangla, 80 in Math and 70 have passed in both subjects in any exam. Express the information by Venn diagram and find how many students failed in both subjects.

Solution : In the Venn diagram, the rectangular region denotes set U of 100 students. and the set of passed students in Bangla and Math are denoted by B and M. So, the Venn diagram is divided into four disjoint sets which are denoted by P, Q, R, F.

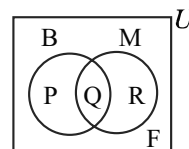
Here, the set of passed students in both subjects $Q = B \cap M$ whose numbers of elements are 70.

$P =$ the set of passed students in Bangla only, whose number of element $= 92 - 70 = 18$

$R =$ the set of passed student in Math only, whose number of elements $= 80 - 70 = 10$

$P \cup Q \cup R = B \cup M$, the set of passed students in one and both subjects, whose number of elements $= 18 + 10 + 70 = 98$

$F =$ the set of students who failed in both subjects, whose number of elements $= 100 - 98 = 2$

$\therefore$ 2 students failed in both subjects.

Exercise 2.1

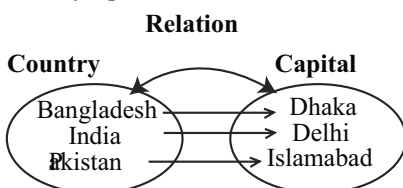
1. Express the following sets in tabular method :
   (a) $\{x \in N : x^2 > 9 \text{ and } x^3 < 130\}$
   (b) $\{x \in Z : x^2 > 5 \text{ and } x^3 \leq 36\}$
   (c) $\{x \in N : x, \text{ factors of 36 and multiple of 6}\}$
   (d) $\{x \in N : x^3 < 25 \text{ and } x^4 < 264\}$
2. Express the following sets in set builder method :
   (a) $\{3, 5, 7, 9, 11\}$ (b) $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$
   (c) $\{4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$ (d) $\{\pm 4, \pm 5, \pm 6\}$
3. If $A = \{2, 3, 4\}$, $B = \{1, 2, a\}$ and $C = \{2, a, b\}$, determine the sets given below:
   (a) $B \setminus C$ (b) $A \cup B$ (c) $A \cap C$ (d) $A \cup (B \cap C)$ (e) $A \cap (B \cup C)$
4. If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{3, 4, 5, 6, 7\}$, justify the followings :
   (*i*) $(A \cup B)' = A' \cap B'$ (*ii*) $(B \cap C)' = B' \cup C'$
   (*iii*) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ (*iv*) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
5. If $Q = \{x, y\}$ and $R = \{m, n, l\}$, find $P(Q)$ and $P(R)$.
6. If $A = \{a, b\}$, $B = \{a, b, c\}$ and $C = A \cup B$, show that the number of elements of $P(C)$ is 2^n, where n is the number of element of C.
7. (a) If $(x-1, y+2) = (y-2, 2x+1)$, find the value of x and y.
   (b) If $(ax - cy, a^2 - c^2) = (0, ay - cx)$, find the value of (x, y).
   (c) If $(6x - y, 13) = (1, 3x + 2y)$, find the value of (x, y).
8. (a) If $P = \{a\}$, $Q = \{b, c\}$ then, find $P \times Q$ and $Q \times P$.
   (b) If $A = \{3, 4, 5\}$, $B = \{4, 5, 6\}$ and $C = \{x, y\}$, find $(A \cap B) \times C$.
   (c) If $P = \{3, 5, 7\}$, $Q = \{5, 7\}$ and $R = P \setminus Q$, find $(P \cup Q) \times R$.
9. If A and B are the sets of all factors of 35 and 45 respectively, find $A \cup B$ and $A \cap B$.
10. Find the set of the number where 31 is the remainder in each case when 346 and 556 are divided by natural numbers.
11. Out of 30 students of any class, 20 students like football and 15 students like cricket. The number of students who like any one of the two is 10. Show with the help of Venn diagram, the number of students who do not like two of the sports.
12. Out of 100 students in any exam, 65% in Bangla, 48% in both Bangla and English have passed and 15% have failed in both subjects.

(a) Express the above information by Venn diagram along with brief description.

(b) Find the numbers who have passed only in Bangla and English.

(c) Find the union of two sets of the prime factors of the numbers who have passed and failed in both subjects.

Relation

We know, the capital of Bangladesh is Dhaka and that of India is Delhi and Pakistan is Islamabad. Here, there is a relation of capital with the country. The relation is country-capital relations. The above relation can be shown in set as follows:

That is, country-capital relation = {(Bangla desh, Dhaka), (India, Delhi), (Pakistan, Islamabad)}

If A and B are two sets, the non-zero subset of R of the Cartesian product $A \times B$ of the sets is called relation of B from A.

Here, R is a subset of $A \times B$ set, that is, $R \subseteq A \times B$.

Example 15. Suppose, $A = \{3, 5\}$ and $B = \{2, 4\}$

$\therefore A \times B = \{3, 5\} \times \{2, 4\} = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

$\therefore R = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$

If the condition is $x > y$, $R = \{(3, 2), (5, 2), (5, 4)\}$

and if the condition is $x < y$, $R = \{3, 4\}$

If an element of set A is x and that of the set B is y and $(x, y) \in R$, we write $x R y$ and read as x *is related to* y. That the element x is R related to element y. Again, if the relation of a set, from set A that is $R \subseteq A \times A$, R is called A related. So, if the relation is given between set A and B, non-zero subset of ordered pair (x, y) with $y \in B$ related to $x \in A$, is a relation.

Example 16. If $P = \{2, 3, 4\}$, $Q = \{4, 6\}$ and $y = 2x$ is relation under consideration between the elements of P and Q, find the relation.

Solution : Given that, $P = \{2, 3, 4\}$ and $Q = \{4, 6\}$

According to the question, $R = \{(x, y) : x \in P,\ y \in Q$ and $y = 2x\}$

Here, $P \times Q = \{2, 3, 4\} \times \{4, 6\} = \{(2, 4), (2, 6), (3,4)(3, 6), (4, 4), (4, 6)\}$

$\therefore R = \{(2, 4), (3, 6)\}$

Required relation is $\{(2, 4), (3, 6)\}$

Example 17. If $A = \{1, 2, 3\}$, $B = \{0, 2, 4\}$ and the relation $x = y - 1$ is under consideration between elements of C and D, find the relation.

Solution : Given that, $A = \{1, 2, 3\}$, $B = \{0, 2, 4\}$

According to the question, relation $R = \{(x, y) : x \in A,\ y \in B \text{ and } x = y - 1\}$

Here, $A \times B = \{1, 2, 3\} \times \{0, 2, 4\}$

$= \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), (2, 4), (3, 0), (3, 2), (3, 4)\}$

$\therefore\ R = \{(1, 2), (3, 4)\}$

> **Activity :** If $C = \{2,5,6\}$, $D = \{4,5\}$ and the relation $x \leq y$ is under consideration between elements of C and D, find the relation.

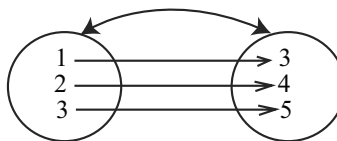
Functions

Let us observe the relation between sets A and B below :

Here, When $y = x + 2$,

$y = 3$ for $x = 1$

$y = 4$ for $x = 2$

$y = 5$ for $x = 3$

That is, for each value of x, only one value of y is obtained and the relation between x and y is made by $y = x + 2$. Hence two variable x and y are so related that for any value of x, only one value of y is obtained even y is called the function of x. The function of x is generally expressed by y, $f(x)$, $g(x)$, $F(x)$ etc.

Let, $y = x^2 - 2x + 3$ is a function. Here, for any single value of x, only one value of y is obtained. Here, both x and y are variables but the value of y depends on the value of x. So, x is independent variable and y is dependent variable.

Example 18. If $f(x) = x^2 - 4x + 3$, find $f(-1)$.

Solution : Given that, $f(x) = x^2 - 4x + 3$

$\therefore\ f(-1) = (-1)^2 - 4(-1) + 3 = 1 + 4 + 3 = 8$

Example 19. If $g(x) = x^3 + ax^2 - 3x - 6$, for what value of a will be $g(-2) = 0$?

Solution : Given that, $g(x) = x^3 + ax^2 - 3x - 6$

$\therefore\ g(-2) = (-2)^3 + a(-2)^2 - 3(-2) - 6$

$= -8 + 4a + 6 - 6$

$= -8 + 4a = 4a - 8$

But $g(-2) = 0$

$\therefore\ 4a-8=0$
or, $4a=8$
or, $a=2$
$\therefore$ if $a=2$, $g(-2)=0$.

Domain and Range

The first set of elements of the ordered pair of any relation is called its domain and the set of second elements is called its range.

Let R from set A to set B be a relation, that is, $R\subseteq A\times B$. The set of first elements included in the ordered pair of R will be domain of R and the set of second elements will be range of R. The domains of R is expressed as Dom R and range is expressed as Range R.

Example 20. Relation $S=\{(2,1),(2,2),(3,2),(4,5)\}$. Find the domain and range of the relation.

Solution : Given that, $S=\{(2,1),(2,2),(3,2),(4,5)\}$

In the relation S, the first elements of ordered pair are $2,2,3,4$ and second elements are $1,2,2,5$.

$\therefore$ Dom $S=\{2,3,4\}$ and Range $S=\{1,2,5\}$

Example 21. If $A=\{0,1,2,3\}$ and $R=\{(x,y): x\in A, y\in A$ and $y=x+1\}$, express R in tabular method and determine Dom R and Range R.

Solution : Given that, $A=\{0,1,2,3\}$ and $R=\{(x,y): x\in A, y\in A$ and $y=x+1\}$

From the stated conditions of R we get, $y=x+1$

Now, for each $x\in A$ we find the value of $y=x+1$.

x	0	1	2	3
y	1	2	3	4

Since $4\notin A$, $(3,4)\notin R$

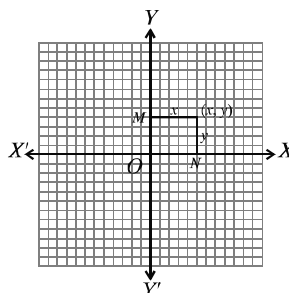
$\therefore\ R=\{(0,1),(1,2),(2,3)\}$

Dom $R=\{0,1,2\}$ and Range $R=\{1,2,3\}$

Activity :
1. If $S=\{(-3,8),(-2,3),(-1,0),(0,-1),(1,0),(2,3)\}$, find domain and range of S.
2. If $S=\{x,y): x\in A, y\in A$ and $y-x=1\}$, where $A=\{-3,-2,-1,0\}$, find Dom. S and Range S.

Graphs :

The diagrammatic view of function is called graphs. In order to make the idea of function clear, the importance of graph is immense. French philosopher and

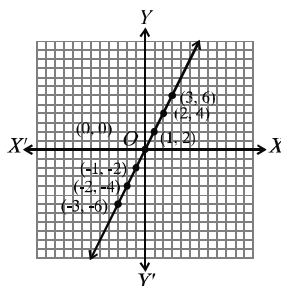
mathematician René Descartes (1596 −1650) at first played a vital role in establishing a relation between algebra and geometry. He introduced the modern way to coplanar geometry by defining the position of point on a plane by two intersecting perpendicular function. He defined the two intersecting perpendicular lines as axes and called the point of intersection origin. On any plane, two intersecting perpendicular straight lines XOX' and YOY' are drawn. The position of any point on this plane can be completely known by these lines. Each of these straight lines are called axis. Horizontal line XOX' is called x axis, Perpendicular line YOY' is called y axis and the point of intersection of the two axes O is called origin.

The number with proper signs of the perpendicular distances of a point in the plane from the two axis are called the Coordinates of that point. Let P be any point on the plane between the two axes. PM and PN are drawn perpendicular from the point P to XOX' and YOY' axes respectively. As a result, $PM = ON$ is the perpendicular distance of the point P from YOY' and $PN = OM$ is the perpendicular distance of P from the XOX'. If $PM = x$ and $PN = y$, the coordinates of the point P is (x, y). Here, x is called abscissa or x coordinate and y is called ordinate or y coordinate.

In Cartesian coordinate, the geometrical figure is shown easily. For this reason, generally we put the independent value along the x axis and the dependent value along the y axis. For some values of independent variables from the domain, we find the similar values of dependent variables and form ordered pair to draw the graphs of the function $y = f(x)$. Then place the ordered pair under (x, y) and joint the obtained points in freehands which is the graph of the function $y = f(x)$.

Example 22. Draw the graph of the function $y = 2x$; where $-3 \leq x \leq 3$.

Solution : In the domain $-3 \leq x \leq 3$, for some values of x, we determine some values of y and form a table :

x	−3	−2 −1	0	1	2	3	
y	−6	−4 −2	0	2	4	6	

On the graph paper, taking the length of square as unit, we identify points of the table on the place and joint the points in free hand.

Example 23. If $f(x)=\frac{3x+1}{3x-1}$, find the value of $\frac{f\left(\frac{1}{x}\right)+1}{f\left(\frac{1}{x}\right)-1}$.

Solution : Given $f(x)=\frac{3x+1}{3x-1}$

$\therefore \quad f\left(\frac{1}{x}\right)=\frac{3\cdot\frac{1}{x}+1}{3\cdot\frac{1}{x}-1}=\frac{\frac{3}{x}+1}{\frac{3}{x}-1} = \frac{-3+x}{3-x}$ [multiplying the numerator and the denomination by x]

or, $\frac{f\left(\frac{1}{x}\right)+1}{f\left(\frac{1}{x}\right)-1}=\frac{-3+x+3-x}{3+x-3+x}$, [By componendo –Dividendo]

$= \frac{6}{2x}=\frac{3}{x}$

Require value is $\frac{3}{x}$

Example 24. If $f(y)=\frac{y^3-3y^2+1}{y(1-y)}$, show that $f\left(\frac{1}{y}\right)=f(1-y)$

Solution : Given, $f(y)=\frac{y^3-3y^2+1}{y(1-y)}$

$\therefore \quad f\left(\frac{1}{y}\right)=\frac{\left(\frac{1}{y}\right)^3-3\left(\frac{1}{y}\right)^2+1}{\frac{1}{y}\left(1-\frac{1}{y}\right)}=\frac{\frac{1-3y+y^3}{y^3}}{\frac{y-1}{y^2}}$

$=\frac{1-3y+y^3}{y^3}\times\frac{y^2}{y-1}=\frac{1-3y+y^3}{y(y-1)}$

again, $f(1-y)=\frac{(1-y)^3-3(1-y)^2+1}{(1-y)\{1-(1-y)\}}$

$=\frac{1-3y+3y^2-y^3-3(1-2y+y^2)+1}{(1-y)(1-1+y)}$

$= \frac{1-3y+3y^2-y^3-3+6y-3y^2+1}{y(1-y)}$

$= \frac{-1+3y-y^3}{y(1-y)} = \frac{-(1-3y+y^3)}{-y(y-1)}$

$= \frac{1-3y+y^3}{y(y-1)}$

$\therefore \ f\left(\frac{1}{y}\right) = f(1-y)$.

Exercise 2·2

1. Which one is the set of factors of 4 ?

   (a) $\{8,16,24...\}$ (b) $(1,2,3,4,8\}$ (c) $\{2, 6, 8\}$ (d) $\{1, 2\}$

2. If a relation of set B from set C is R, which one of the following is right ?

   (a) $R \subset C$ (b) $R \subset B$ (c) $R \subseteq C \times B$ (d) $C \times B \subseteq R$

3. If $A = \{6, 7, 8, 9, 10, 11, 12, 13\}$, answer the following questions :

   (i) Which one is builder method of set A ?

   (a) $\{x \in N: 6 < x < 13\}$ (b) $\{x \in N: 6 \leq x < 13\}$

   (c) $\{x \in N: 6 \leq x \leq 13\}$ (d) $\{x \in N: 6 < x \leq 13\}$

   (ii) Which one is the set of prime numbers ?

   (a) $\{6, 8, 10, 12\}$ (b) $\{7, 9, 11, 13\}$ (c) $\{7, 11, 13\}$ (d) $A = \{9, 12\}$

   (iii) Which is the set of multiple of 3 ?

   (a) $\{6, 9\}$ (b) $\{6, 11\}$ (c) $\{9, 12\}$ (d) $\{6, 9, 12\}$

   (iv) Which is the set of factor of greater even number ?

   (a) $\{1, 13\}$ (b) $\{1, 2, 3, 6\}$ (c) $\{1, 3, 9\}$ (d) $\{1, 2, 3, 4, 6, 12\}$

4. If $A = \{3, 4\}$, $B = \{2, 4\}$, find the relation between elements of A and B considering $x > y$.

5. If $C = \{2, 5\}$, $D = \{4, 6\}$ and find the relation between element of C and D considering relation $x + 1 < y$.

6. If $f(x) = x^4 + 5x - 3$, the find value of $f(-1)$, $f(2)$ and $f\left(\frac{1}{2}\right)$.

7. If $f(y) = y^3 + ky^3 - 4y - 8$, for which value of k will be $f(-2) = 0$.

8. If $f(x) = x^3 - 6x^2 + 11x - 6$, for which value of x will be $f(x) = 0$.

9. If $f(x)=\dfrac{2x+1}{2x-1}$, find the value of $\dfrac{f\left(\dfrac{1}{x^2}\right)+1}{f\left(\dfrac{1}{x^2}\right)-1}$.

10. If $g(x)=\dfrac{1+x^2+x^4}{x^2}$, show that, $g\left(\dfrac{1}{x^2}\right)=g(x^2)$

11. Find the domain and range of the following relations :

(a) $R=\{(2,1),(2,2),(2,3)\}$ (b) $S=\{(-2,4),(-1,1),(0,0),(1,1),(2,4)\}$

(c) $F=\left\{\left(\dfrac{1}{2},0\right),(1,1),(1,-1),\left(\dfrac{5}{2},2\right),\left(\dfrac{5}{2},-2\right)\right\}$

12. Express the relations in tabular method and find domain and range of following relations :

(a) $R=\{(x,y): x\in A,\ y\in A \text{ and } x+y=1\}$ where $A=\{-2,-1,0,1,2\}$

(b) $F=\{(x,y): x\in C,\ y\in C \text{ and } x=2y\}$ where $C=\{-1,0,1,1,3\}$

13. Plot the points $(-3,2),\ (0,-5),\ \left(\dfrac{1}{2},-\dfrac{5}{6}\right)$ on graph paper.

14. Plot the three points $(1,2),\ (-1,1),\ (11,7)$ on graph paper and show the three points are on the same straight line.

15. Universal set $U=\{x: x\in N \text{ and } x \text{ is an odd number}\}$

$A=\{x\in N: 2\le x\le 7\}$

$B=\{x\in N: 3<x<6\}$

$C=\{x\in N: x^2>5 \text{ and } x^3<130\}$

(a) Express A in tabular method.

(b) Find A' and $C-B$.

(c) Find $B\times C$ and $P(A\cap C)$.